Rectilinear Coordinate Frames for Deep Sea Navigation

WOODS HOLE OCEANOGRAPHIC INSTITUTION 1930

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SeaBED
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The Gist

Since 1964, the National Deep Submergence Facility and Deep Submergence Laboratory at the Woods Hole Oceanographic Institution (WHOI) have performed thousands of scientific dives with human occupied, remotely operated, autonomous, and towed vehicles. Each of these vehicles uses an equirectangular map projection within their navigation systems, colloquially known as the AlvinXY coordinate scheme.

Equirectangular projections provide a simple, affine mapping between geographic coordinates and the coordinates of a cartesian grid. However, AlvinXY and similar coordinate systems disregard the effect of depth upon the coordinate mapping. Advances in underwater navigational instrumentation during the past forty years now allow localization to such precision that, in some cases, the precision of the navigation solution is comparable to that of errors introduced by working in this simple coordinate space.

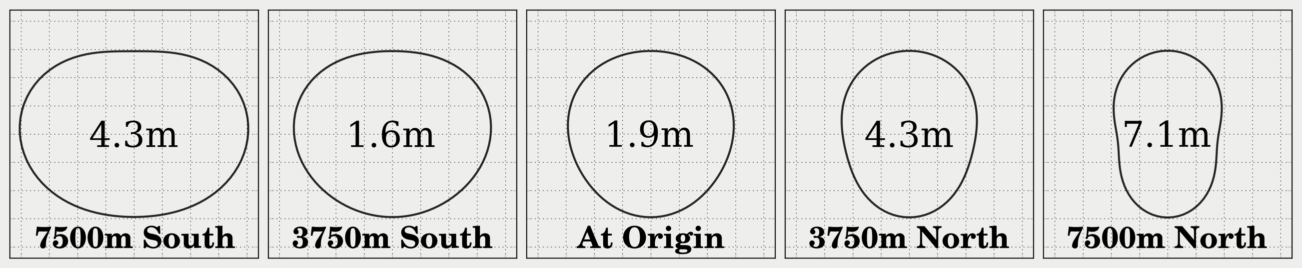
In particular, these coordinate schemes can introduce large errors when coupled with range-based navigation schemes such as Long Baseline Navigation (LBL). These errors can be on the order of meters, or even tens of meters, for a typical LBL survey.

How / Why?

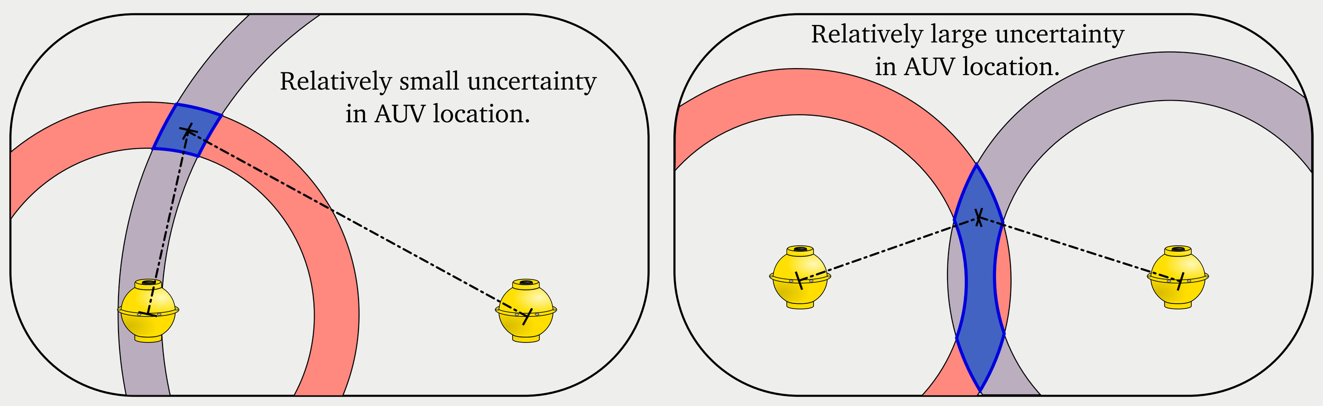
Equirectangular coordinate schemes make a number of assumptions about the conversion between a local "XY" projected coordinate frame, and geodetic coordinates (latitude and longitude). These assumptions include, in approximate order of error magnitude:

- **Ellipsoidal Approximation**
- **Rectilinearity (East ⊥ North)**
- **Ignoring Depth Effects**

These approximations, particularly the ignorance of depth effects, lead to significant errors in calculated ranges, and distort "circles" of range into non-circular projected shapes. Projected 4km range circles and maximum error for several distances north or south of a 50°N equirectangular origin are shown below, with radial error exaggerated 50 times in the sketch.



Radial error in range circles can lead to significant error in computed vehicle locations using LBL. As shown below, this error is magnified near the baseline between two ranging beacons. The rest of the figures on this page show specific calculated range and location errors for an AUV expedition at 85°N latitude.



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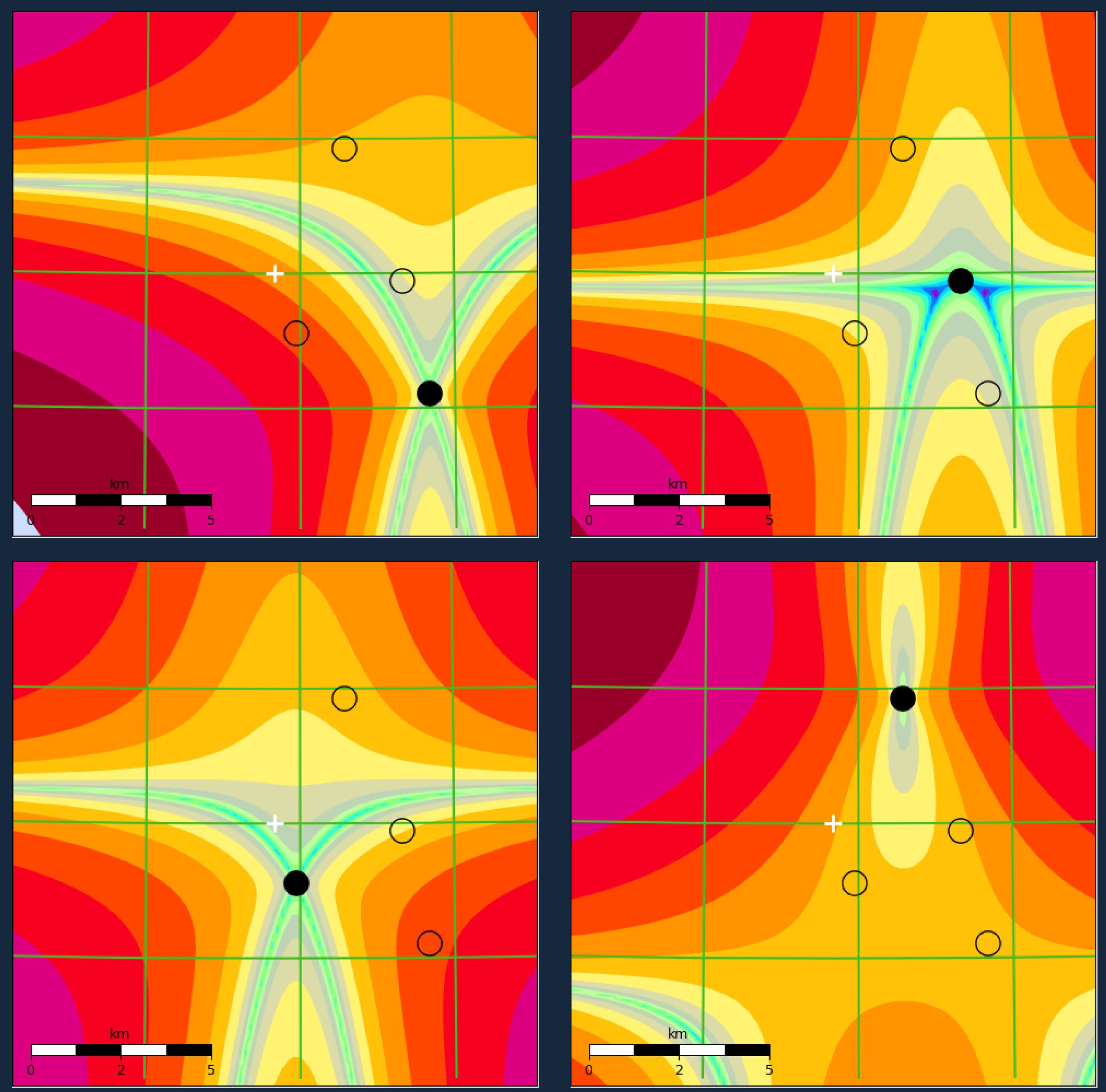
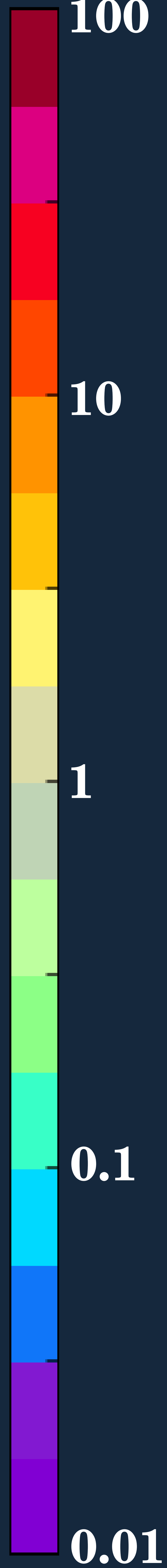
Range Error in Meters

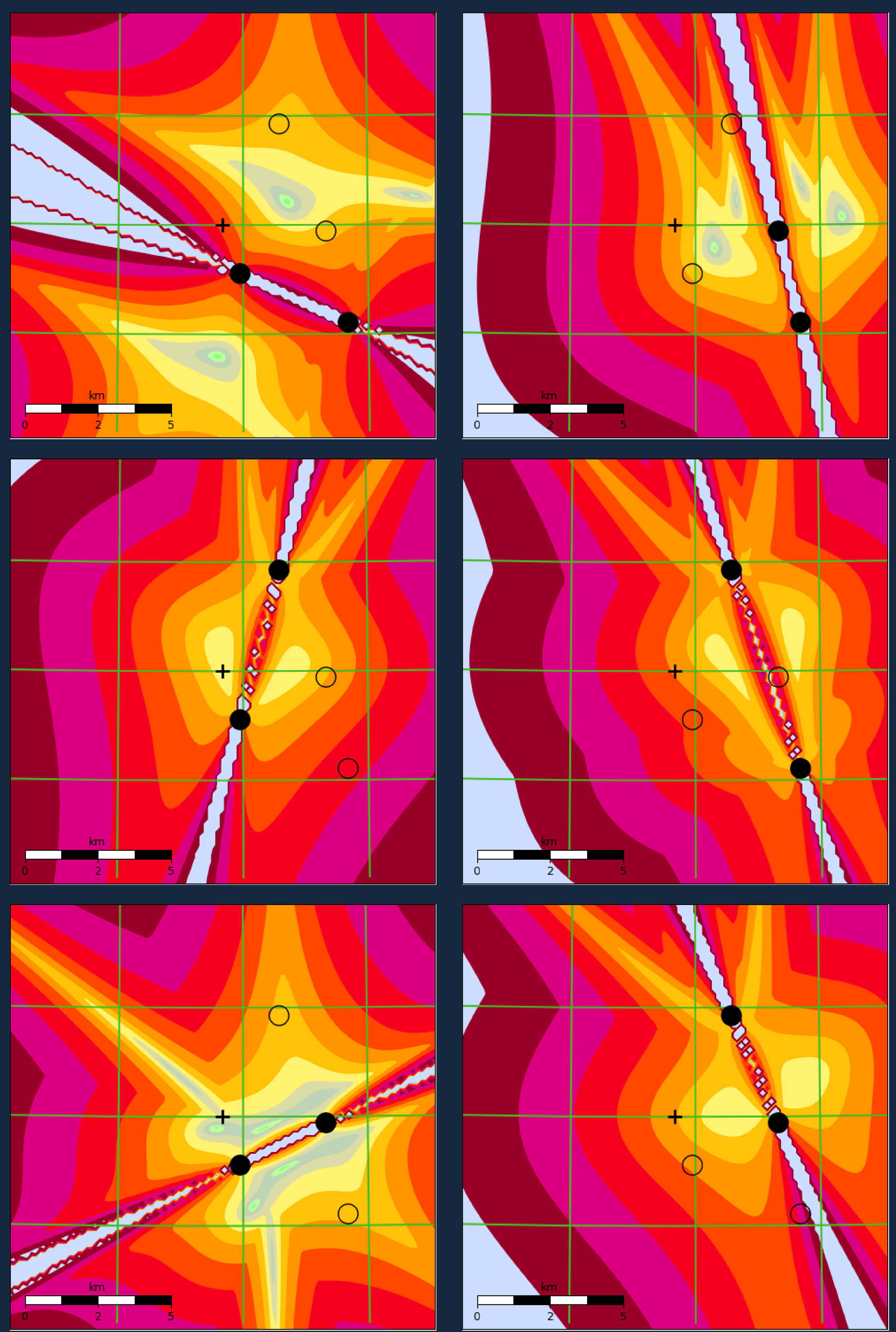




Key

- ✛ ✛ Equirectangular Origin
- ● ● Active LBL Beacon
- ○ ○ Unused LBL Beacon

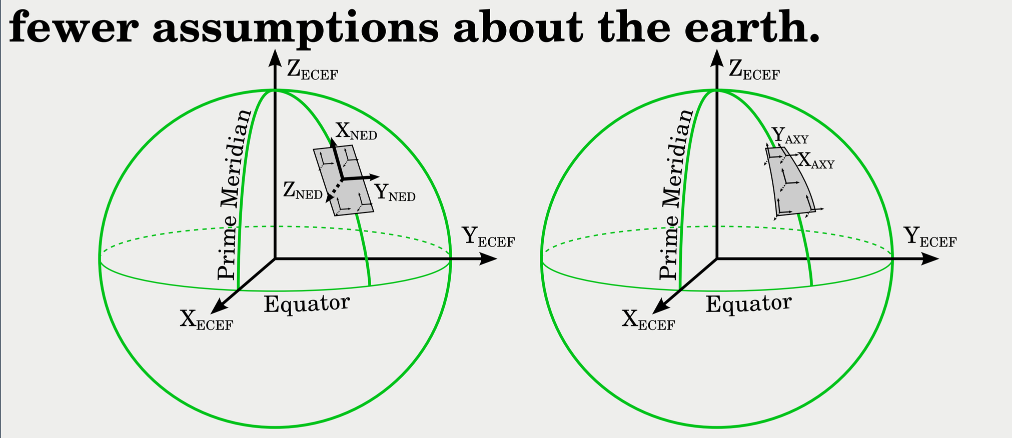
Location Error in Meters



So What?

For much of AlvinXY's history it has been a catch-all tool for navigation and mapping. As navigation accuracy and precision continue to improve, a similar catch-all solution will be difficult to find. Map projections all trade off navigation, path planning, estimation, and operational benefits. UTM averages error across six degrees of longitude. Local Transverse Mercator projections have relatively low error across a large survey area, but still require that depth be taken into consideration.

Local Tangent Plane approximations provide one possible solution. North-East-Down coordinate frames are routinely used in aerial surveys, and make fewer assumptions about the earth.



North-East-Down Local Tangent Plane
Linear transformation of ECEF.

AlvinXY Grid Coordinates
Axis orientation varies across grid.

Symbolic Equations

$$l = \frac{\pi a(1-e^2)}{180(1-e^2\sin^2\theta)^{\frac{3}{2}}}$$

$$L = \frac{\pi a \cos\theta}{180\sqrt{1-e^2\sin^2\theta}}$$

Binomial Approximation

$$l = \frac{\pi a \cos\theta}{180}\sum_{k=0}^{N-1}\left[\frac{e^{2k}\sin^{2k}\theta}{k!}\prod_{n=0}^{k-1}\left(-\frac{1}{2}+k-n\right)\right]$$

$$L = \frac{\pi a(1-e^2)}{180}\sum_{k=0}^{N-1}\left[\frac{e^{2k}\sin^{2k}\theta}{k!}\prod_{n=0}^{k-1}\left(\frac{1}{2}+k-n\right)\right]$$

For Various Ellipsoids

Clarke 1866

This (antiquated) ellipsoid is used by AlvinXY and elsewhere in the oceanographic community.

$$l = 111132.09 - 566.05\cos(2\lambda) + 1.20\cdot\cos(4\lambda) - 0.002\cdot\cos(6\lambda)$$

$$L = 111415.13\cdot\cos(\lambda) - 94.55\cdot\cos(3\lambda) + 0.12\cdot\cos(5\lambda)$$

WGS84

The GPS system is based on this ellipsoid.

$$l = 111132.952 - 559.849\cos(2\lambda) + 1.175\cdot\cos(4\lambda) - 0.0023\cdot\cos(6\lambda)$$

$$L = 111412.877\cdot\cos(\lambda) - 93.503\cdot\cos(3\lambda) + 0.117\cdot\cos(5\lambda)$$